

EVOLUTIONARY OPTIMIZATION IN DYNAMIC FITNESS LANDSCAPE ENVIRONMENTS

Aragón V. S., Esquivel S. C.,
Laboratorio de Investigación y Desarrollo en Inteligencia Computacional (LIDIC)¹
Facultad de Ciencias Físico, Matemática y Naturales
Universidad Nacional de San Luis
Ejército de los Andes 950 – Local 106
5700 – San Luis Argentina.
{vsaragon, esquivel}@unsl.edu.ar

ABSTRACT

Non-stationary, or dynamic, problems change over time. There exist a variety of forms of dynamism. The concept of dynamic environments in the context of this paper means that the fitness landscape changes during the run of an evolutionary algorithm.

Genetic diversity is crucial to provide the necessary adaptability of the algorithm to unexpected changes.

Two key concepts to maintain genetic diversity in the population are incorporated to the algorithm and proposed here: macromutation operators and random immigrants.

The algorithm was tested on a set of dynamic testing functions provided by a dynamic fitness problem generator. The main goal was to determine the algorithm ability to face changes and dimensional or multimodal scalability in the functions.

The effectiveness and limitations of the proposed algorithm in diverse scenarios of a dynamic environment is discussed from results empirically obtained.

Keywords: Evolutionary computation, dynamic fitness landscape, macromutation, random immigrants, multi-modal optimization.

¹ The LIDIC is supported by the Universidad Nacional de San Luis and the ANPCYT (National Agency to Promote Science and Technology).

1. INTRODUCTION

In general the conditions of an optimization problem changes by one of the following reasons or a combination of both [1]: 1) The objective function changes itself, 2) The constraints change. A change in the objective function appears when the purpose of the problem changes. Here conditions which were considered desirable before can turn out to be undesirable now and vice versa. Changes in constraints, which modify feasibility of solutions, are related to resources and their availability. Changes can be small or big, soft or abrupt, chaotic, etc. When changes are big, abrupt or chaotic the similarity between solutions found so far and the new ones can be worthless. Even under these hard environments Evolutionary Computation (EC) offers advantages, which are absent in other heuristics when searching for solutions to non-stationary problems. The main advantage relies in the fact that Evolutionary Algorithms (EAs) keep a population of solutions. Consequently, facing the change, they allow moving from a solution to another one to determine if any of them are of merit to continue the search from them instead of from scratch [2]. Goldberg and Smith [3], Cobb[4] and Grefenstette [5] initiated the research related to the behaviour of EAS on dynamic fitness functions between 1987 and 1992. Recently the interest in this area was dramatically incremented [6], [7], [8], [9], [10], [11], [12], [13], [14], and [15]. The following sections are organized as follows. Section 2 presents a definition of dynamic environments studied in this work. Section 3 describes the dynamic test functions used. Section 4 describes the EA. In section 5 the experiments performed are described. In section 6 results are discussed and finally this document shows our conclusions, current and future work.

2. DYNAMIC FITNESS PROBLEM DEFINITION

A general definition, which describes and characterizes a dynamic fitness function, is introduced here. The approach we follow assumes that each dynamic function consists of a base static function and a sequence of dynamic functions obtained from the base function and the application of a set of dynamic rules.

Definition 1:

Let be Ψ the searching space, a vector $\bar{x} \in \Psi$ and the time $t \in N$. A dynamic fitness function DF^t is defined as follows :

$$DF^t = \begin{cases} fb^t(\bar{x}) & \text{if } t = 0 \\ g^t(DF^{t-1}(\bar{x}), ch^t, s^t) & \text{otherwise} \end{cases} \quad (I)$$

where $fb^0(\bar{x})$ is the base static function with m possible features to be modified.

Function g^t is a function having as arguments the dynamic fitness function at time $t-1$, a set ch^t of all possible changes to be applied to that function at time t and a given severity of the change, and returns a new fitness function at time t .

Definition 2: Function g^t is more precisely defined as follows:

$$g^t(DF^{t-1}(\bar{x}), ch^t, s^t) = \text{apply_changes_on_}DF^{t-1}(ch^t, L)$$

where the first argument of $\text{apply_changes_on_}DF^{t-1}$:

$$ch^t = \bigcup_{i=1}^m \bigcup_{j=1}^n \{c_i, FL_j\}$$

is an ordered set of size $m \times n$ of all possible changes at time t . When building this set, c_i indicates which function characteristic changes and FL_j indicates which part of the DF^{t-1} landscape will be modified within the i^{th} characteristic.

The second argument of $\text{apply_changes_on_}DF^{t-1}$, L is a binary vector of length $m \times n$, which depending on s^t indicates which members of ch^t are selected and which are not selected to modify DF .

3. DYNAMIC TESTING FUNCTIONS

In this section we will see that the functions provided by the Test problem Generator [16] belong to the dynamic functions defined in (I).

In this case we have:

$\Psi = R^n$ the searching space, a vector $\bar{x} \in R^n$ and the time $t \in N$.

The dynamic function DF^t defined as in (I) with the base static function defined as:

$$fb^t(\bar{x}) = \max_{i=1,k} \left[H_i^t - R_i^t * \sqrt{(x_1 - x_{1i}^t)^2 + \dots + (x_k - x_{ni}^t)^2} \right]$$

which specifies a “field of cones”, where k indicates the number of cones in the environment and each cone is independently specified by its location $(x_{1i}, x_{2i}, \dots, x_{ni})$, its height H_i , and its slope R_i .

The components of the vector defining the cone location are $x_{ij} \in [-1, 1]$.

Each of these cones is combined together by means of the \max function. Each time it is called the generator creates a randomly generated morphology. The user specifies the range of random values for the height, slope and location of cones:

$H_i \in [Hbase, Hbase + Hrange]$, $R_i \in [Rbase, Rbase + Rrange]$ and $x_{ij} \in [-1, 1]$.

In this case, when building the set ch^t of all possible changes, a characteristic $c_i \in \{\text{height, slope, location}\}$ ($m = 3$) and FL_j indicates which of the k cones of the DF^{t-1} landscape will be modified within the i^{th} characteristic. Consequently, when $\text{apply_changes_on_}DF^{t-1}$ operates, the corresponding modifications in DF will be done only for those characteristics in the cones where $L_{j+k(i-1)} = 1$, in the binary vector L , for $1 \leq i \leq m$, $1 \leq j \leq k$.

In this way we can see that the generator allows changing height, slope and location of one or more cones in the field of cones, which represents the morphology of the fitness landscape.

The severity of changes has many degrees, such that degree 1.0 corresponds to soft and/or small changes and degree 4.0 to chaotic changes. The degree of severity is calculated by using the logistic function:

$$s^t = Y_p = A * Y_{p-1} * (1 - Y_{p-1})$$

where A is a constant indicating the degree of severity of the characteristic (height, slope, or location) that will be modified and Y_p is the value of the logistic function at iteration p . For more details see [16].

4. EVOLUTIONARY ALGORITHM

We chose an evolutionary algorithm which combines and exploit various forms of macro-mutation similar to both, the hyper-mutation and random immigrants initially designed by Grefenstette [5].

4.1. REPRESENTATION

The population P is composed of a constant number N of chromosomes. Each individual consists of a single chromosome, where each gene is a real value in the interval $[-1.0, 1.0]$ representing a coordinate in the search space. That is, the i^{th} individual in the population P is represented by the chromosome:

$$\mathbf{P}^i = \langle x_{i1}, x_{i2}, \dots, x_{il} \rangle$$

where x_{ij} denotes the j^{th} coordinate of the i^{th} individual with $j = 1, \dots, l$ and l is the chromosome length.

4.2. OPERATORS

Conventional operators:

Selection: Couples of parents for the mating pool are selected by means of proportional selection.

Recombination: The conventional one point crossover is used to exchange genetic material between parents. The operator is applied with a P_{cross} probability.

Mutation: Uniform mutation is used and it is applied with a P_{mut} probability. When an individual undergoes mutation, each gene has exactly the same chance of undergoing mutation. As a result the mutated gene has a new allele value randomly chosen from the domain of the corresponding parameter (vector component).

Specialized operators for macromutation:

Recrudescence: This operator increments the probabilities of undergoing recombination and/or mutation for a part of the population. It is applied in every generation with a probability P_{recru} and produces a radical genotypic reorganization on the individuals where it is applied. These individuals are selected randomly with uniform probability.

Crisis: Acts as the recrudescence operator but it is applied on the whole population at determined intervals during the initial stages of the evolution.

Random immigrants: A percentage $z\%$ of the population is replaced by individuals randomly generated.

4.3. EVOLUTIONARY ALGORITHM PSEUDOCODE

The structure of the proposed evolutionary algorithms follows:

```

0.  $t = 0$  /* initial generation */
1. Generate  $fb^t$  function and set  $DF^t = fb^t$ 
2. Initialise  $P^t$  /* initial population */
3. Evaluate  $P^t$ 
4. while (actual_number-changes  $\leq$  total_number-changes) do
5. {
6.    $t = t + 1$ 
7.   if (crisis) and ( $t < \text{number\_gen\_with\_crisis}$ ) then
8.     Apply_crisis_operator
9.     Generate next population  $P'^t$  using traditional operators and recrudescence if appropriate
10.    Evaluate  $P'^t$ 
11.    Calculate_statistics of  $P'^t$ 
12.    Remember_the_best_of_generation /* elitism */
13.    if (function_changes) then
14.    {
15.      Store_statistical_report
16.      Build_vector_L
17.      Apply_changes_on_ $DF^{t-1}$ (ch, L) and obtain new  $DF^t$ 
18.    }
19.    if (occured_changes) then
20.    { Evaluate  $P'^t$  with new  $DF^t$ 
21.      Calculate_statistics of  $P'^t$ 
22.      Remember_the_best_of_generation /* elitism */
23.      Apply_crisis_operator
24.    }
25.    if (finish_apply_crisis) then
26.    {
27.      apply_random_inmigrants_operator
28.      Evaluate  $P'^t$ 
29.      Calculate_statistics of  $P'^t$ 
30.      Remember_the_best_of_generation /* elitism */
31.    }
32.    Let  $P^t = P'^t$ 
33.  } /* end while */
34. Report_statistics

```

In line 13, function_changes is responsible to detect if a change in dynamic fitness function must occur. In our case changes occur at constant intervals, then this function only verifies if the generation number corresponds to one where the change must occurs.

If a change must occur we store in the L vector what changes are to be done and in which cones on the landscape to apply them (see line 16). Then the `apply_changes` function obtains a new dynamic fitness function in line 17.

In the line 22, `occured_function` tests if a change effectively had occurred, in which case the application of macromutation operators creates the necessary genetic diversity. Then a new generation begins and so on, until the end condition is reached.

5. EXPERIMENTS DESCRIPTION

5.1. PARAMETERS OF THE EVOLUTIONARY ALGORITHM

The parameter settings for the EA remain fixed throughout all experiments and all scenarios, and were determined as the best after some initial trials:

The population size $|P|$ was set to 100 individuals. P_{cross} and P_{mut} were fixed at 0.25 and 0.5, respectively. For *recrudescence*, P_{recru} was set to 0.2, and the augmented probabilities of crossover and mutation were fixed at 0.5 and 0.8, respectively. The *crisis* operator is applied to the 10% of the number of generation, between two consecutive changes in the environment. The percentage of random immigrants was fixed at 30% of the population. Immigrants are inserted when a change was detected and after the application of the crisis operator. This decision prevents that new immigrants be affected by the crisis operator. The individuals to be replaced by immigrants are randomly selected with equal probability. A number of experiments were designed differing in the function selected and the changes to perform on it. For each of these experiments 30 runs were performed with distinct initial population. Values in tables of section 6 are mean values.

5.2. PARAMETERS OF THE FUNCTION GENERATOR

Table 1 shows the parameter settings for the generator for all functions.

Hbase	Hrange	Rbase	Rrange	A
30	70	1	12	3.3

Table 1. Parameter Settings for the Generator

The constant A is used by the logistic function to determine the change severity. The value chosen for A creates a severity of degree between median and large (near to the upper limit of 4.0 required by the simulator).

We worked on 5 different functions whose features of dimensionality and multimodality (number of cones) are indicated in table 2.

Function	dim-#cones	dim-#cones	dim-#cones	dim-#cones	dim-#cones	dim-#cones
f_1	2-5	2-30	5-5	5-30	10-5	10-30
f_2	2-5	2-30	5-5	5-30	10-5	10-30
f_3	2-5	2-30	5-5	5-30	10-5	10-30
f_4	2-5	2-30	5-5	5-30	10-5	10-30
f_5	2-5	2-30	5-5	5-30	10-5	10-30

Table 2. Functions used in the experiments

Because the generator randomly creates the functions, we adopted the following working methodology. For example for function $f1$ (see table 3), first we select heights (H) and slopes (R) with the greatest multimodality. C indicates the cone identifier.

C	H	R	C	H	R	C	H	R
1	0.000000	0.000000	11	61.556893	5.811603	21	86.834253	6.018287
2	39.254461	13.425523	12	93.957383	2.548734	22	46.736654	7.188162
3	88.798945	2.201534	13	60.579642	5.152966	23	48.702074	10.149014
4	75.510408	3.834928	14	89.383742	8.525519	24	36.545954	11.617667
5	30.118679	12.300053	15	46.533252	10.423104	25	91.210267	2.415192
6	57.280549	13.325354	16	35.963361	2.647181	26	54.656304	3.071827
7	90.701662	12.850853	17	84.346277	5.990322	27	39.453273	5.902942
8	69.366403	12.017908	18	57.764563	3.513832	28	71.159464	5.190918
9	83.719877	6.973150	19	31.260441	13.380469	29	86.679211	8.862785
10	99.706369	2.875522	20	32.983730	12.417660	30	49.167685	7.235978

Table 3. Initial Heights and Slopes for f_{1b}^0

A similar table, not shown here for space limitations, is built for the 10 coordinates of each cone (the greatest dimensionality) the data associated with the experiments are available for any interested reader.

When scalability is to be modified, to work with lower dimensions and lesser number of cones we obtain the required values from these tables. For example if we wish to work with $f1$ for 2-5, the heights and slopes of the first five cones are retrieved from the tables only for the first two dimensions of the tables of coordinates. Analogously we proceeded with the remaining functions.

This working methodology allowed us to study the adaptability of the algorithm to changes and its behaviour when facing scalability in space dimension and number of cones.

Table 4 shows how, depending on dimensionality and multimodality, the intervals between changes were fixed for all functions, the main goal here was to locate the optimum with an acceptable error.

dim-#cones	Generations between changes
2-5	350
2-30	900
5-5	1000
5-30	3500
10-5	5000
10-30	10000

Table 4. Change intervals.

Each time the algorithm run as many generations as changes were desired to make. For all experiments we fixed at 4 the number of changes. For example, for each of the functions with dimensionality-multimodality equal to 2-5 a total of 1400 generations were needed.

5.3. TYPES OF CHANGE

Four scenarios were designed, each representing a type of change.

Scenario 1: Change in the height of all cones.

Scenario 2: Change in the height of the cone containing the optimum value.

Scenario 3: Change in the location of all cones.

Scenario 4: Change in the location of the cone containing the optimum value.

Also, experimentation was conducted with changes in the slope of one and all cones, but results on these scenarios were very similar to those of scenarios 3 and 4. For this reason they are not shown here.

5.4. PERFORMANCE METRICS

To measure precision and adaptability of the algorithm the following metrics were used:

Precision [17]: It is a metric specifically developed for non-stationary environments. It measures the average difference, between the best individual in the population at the generation “just before the change” and the optimal value, averaged to the number of changes. More precisely:

$$\text{Precision } (P) = \frac{1}{K} \sum_{i=1}^k (opt - b_i)$$

where:

K is the number of changes suffered by the fitness function.

opt is the mean value of optimal values found in each change.

b_i is the best value found before the i^{th} change.

Adaptability [17]: It is the difference between the value of the best individual found at each generation and the average optimal value through the whole run. It is defined as:

$$\text{Adaptability}(A) = \frac{1}{K} \sum_{i=1}^k \frac{1}{t} \sum_{j=0}^{t-1} (opt_i - b_j)$$

where:

K is the number of changes suffered by the fitness function.

opt_i is the optimal value found after the i^{th} change.

b_j is the best value found in the j^{th} generation after the last change.

t is the amount of generations between two consecutive changes.

From definitions it is clear that small values of the metrics P and A indicate better results. In particular a zero value for adaptability indicates that the algorithm finds the optimum before the landscape morphology changes. On the other side, a zero value for precision means that the best individual in the population is found as the global optimum in every generation.

6. RESULTS

Tables 5 and 6 summarize the results obtained. In these tables each entry indicates the number of runs where the algorithm detected 100% or (at least) 75% of the changes. Consequently, *CC* indicates the percentage of changes detected by the algorithm for each function. At the bottom of these tables, *PA* and *AA* indicate the average mean values of precision and adaptability over all five functions.

<i>f</i>	<i>CC</i>	2 – 5 Scenarios				5 - 5 Scenarios				10 - 5 Scenarios			
		1	2	3	4	1	2	3	4	1	2	3	4
<i>f1</i>	100%	30	30	30	30	30	30	30	30	1	30	30	30
	75%	0	0	0	0	0	0	0	0	29	0	0	0
<i>f2</i>	100%	30	30	30	30	30	30	30	30	30	9	30	30
	75%	0	0	0	0	0	0	0	0	0	21	0	0
<i>f3</i>	100%	30	30	30	30	30	30	30	30	0	0	30	30
	75%	0	0	0	0	0	0	0	0	30	30	0	0
<i>f4</i>	100%	30	30	30	30	30	30	30	30	30	30	30	30
	75%	0	0	0	0	0	0	0	0	0	0	0	0
<i>f5</i>	100%	30	27	30	30	25	30	30	30	6	0	30	30
	75%	0	3	0	0	5	0	0	0	24	30	0	0
<i>PA</i>		.0316	.0300	.1196	.0269	.8967	.7631	.7059	.7066	3.081	2.959	2.695	2.73
<i>AA</i>		.0595	.0504	.0562	.0556	1.110	.9532	.9209	.9139	3.405	3.261	3.022	3.08

Table 5. Percentage of changes detected, mean and average mean values for the performance metrics on 5 cones landscapes, dimensionally scaled

<i>f</i>	<i>CC</i>	2 – 30 Scenarios				5 - 30 Scenarios				10 - 30 Scenarios			
		1	2	3	4	1	2	3	4	1	2	3	4
<i>f1</i>	100%	30	29	30	30	30	30	30	30	29	0	30	30
	75%	0	1	0	0	0	0	0	0	1	30	0	0
<i>f2</i>	100%	30	30	30	30	18	30	30	30	0	30	30	30
	75%	0	0	0	0	2	0	0	0	30	0	0	0
<i>f3</i>	100%	30	30	30	30	16	30	30	30	30	6	30	30
	75%	0	0	0	0	14	0	0	0	0	24	0	0
<i>f4</i>	100%	30	30	30	30	30	5	30	30	30	0	30	30
	75%	0	0	0	0	0	25	0	0	0	30	0	0
<i>f5</i>	100%	30	30	30	30	30	17	30	30	30	30	30	30
	75%	0	0	0	0	0	13	0	0	0	0	0	0
<i>PA</i>		.1945	.0238	.0221	.0232	.6458	.7121	.7935	1.291	2.749	2.103	2.041	2.01
<i>AA</i>		.0413	.0467	.0421	.0415	.8442	.9242	1.075	1.079	3.124	2.361	2.296	2.29

Table 6. Percentage of changes detected, mean and average mean values for the performance metrics on 30 cones landscapes, dimensionally scaled

From a general analysis of both tables it come out that the harder scenarios for the EA are scenarios 1 and 2 (change in the height of all cones and the change in the height of the cone containing the optimum value, respectively). The hardness of these changes resides in the fact that when they happen, not only modify the height of the cone containing the optimum but also it can occur that this cone does not contain the optimum any more. Consequently, there is a simultaneous combined effect: change in the height of the cone that contains the optimum and its location. Even though, for some functions in this hard scenarios the algorithm detect 100% of the changes performed through

the 30 runs, and in the worst case it is able to detect 75% of the changes through the 30 runs, with precision ranging from 0.0 (very good) to 3.4 (acceptable).

Scenarios 3 and 4 resulted easy for the EA, because in all functions 100% of the changes were detected.

6.1. SCALABILITY ANALYSIS AT DIMENSIONALITY LEVEL

Tables 5 and 6 indicate that, maintaining fixed the number of cones, when the dimensionality augments the algorithm performance degrades in the hard scenarios and for most functions. Here we can observe that CC decays from 100% to 75%.

6.2. SCALABILITY ANALYSIS AT MULTIMODALITY LEVEL

By contrasting tables 5 and 6, we can see that regarding the number of changes detected (100% or 75%) for a given dimensionality, the behaviour of the algorithm is almost similar.

Regarding P we see that an increment on the values of the metric are in correspondence with an increment in dimensionality or in multimodality. This fact shows that the algorithm not always succeeded to adapt itself to the 100% of the changes produced.

7. CONCLUSIONS

Results obtained by the proposed algorithm are promising when compared with those from previous evolutionary approaches to dynamic environments [2]. The presented algorithm is less memory and time consuming.

In the worst case, for some functions and harder scenarios (higher dimensionality and multimodality), the algorithm is not successful to adapt to 100% of the changes (120 changes in 30 runs). But indeed, under these conditions, it is able to detect at least 75% of the changes (90 changes) produced in 30 runs.

In order to improve the performance of the algorithm under the hardest conditions, issues related to self-adaptation of operator probabilities will be considered. Presently we are working simultaneously in two problems: automatic detection of changes and an investigation to determine if the algorithm is able to follow changes produced in very short intervals. Under this scenario the important issue is not the precision achieved by the algorithm but its ability of creating at least one individual following the course towards the optimum.

8. ACKNOWLEDGEMENTS

We acknowledge the co-operation of the project group for providing new ideas and constructive criticisms. Also to the Universidad Nacional de San Luis and the ANPCYT from which we receive continuous support.

9. BIBLIOGRAPHY

[1] Michalewicz, Z. and Fogel, D.B., - "How to Solve It: Modern Heuristics", Springer, 2000

- [2] Branke, J. - “ Evolutionary Optimization in Dynamic Environments”, Kluwer Academic Publishers, 2002.
- [3] Golberg, D. E. and Smith, R. E. – “Nonstationary Function Optimization using Genetic Algorithms with Dominance and Diploidy”, Proceedings of the Second International Conference on Genetic Algorithms, pp. 59-68, Lawrence Erlbaum Associates, 1987.
- [4] Cobb, H.G. – “An Investigation into the use of Hypermutation as an Adaptive Operator in Genetic Algorithms having Continuous, Time_Dependent Non-Stationary Environmentes”, Technical Report 6760, Naval Research Laboratory, USA, 1990.
- [5] Grenfenstette, J. J. – “Genetic Algorithms for Changing Environments”, Proceedings of the Second Conference on Parallel Problem Solving from Nature”, pp. 137 – 144, North Holland, 1992.
- [6] Mori, N. S. et al. – “Adaptation to Changing Environments by means of the Memory Based Thermodynamic Genetic Algorithm”, Proceeding of the Seventh International Conference on Genetic Algorithms, pp 299 – 306, Morgan Kaufmann, 1997.
- [7] Lewis, J., Hart, E. and Ritchie, G. – “ A Comparision of Dominance Mechanism and Simple Mutation in Non-Stationary Problems”, Proceeding of the Seventh International Conference on Genetic Algorithms, pp 138-148, Morgan Kaufmann, 1997.
- [8] Bäck, T. – “On the Behaviour of Evolutionary Algorithms in Dynamic Fitness Landscapes”, Proceeding of IEEE International Conference on Evolutionary Algorithms, pp 446-451, IEEE Service Centre, 1998.
- [9] Wile, C. O. - “Evolutionary Dynamics in Time-Dependent Environments”, PhD. Thesis, Institut für Neuroinformatik, Ruhr-Universität, Bochum, Germany, 1999.
- [10] Trojanoswky, K. and Michalewicz, Z. – “Searching for Optima in Non-Stationary Environments”, Proceeding of Congress on Evolutionary Computation, pp 1843-1850, IEEE Service Centre, 1999.
- [11] Wicker, K. and Weicker, N. – “ On Evolutionary Strategy Optimization in Dynamic Environments”, Proceeding of Congress on Evolutionary Computation, pp 2039-2046, IEEE Service Centre, 1999.
- [12] Liles, W. and De Jong, K. – “The Usefulness of Tag Bits in Changing Environments”, Proceeding of Congress on Evolutionary Computation, pp 2054-2060, IEEE Service Centre, 1999.
- [13] Yi, W., Lin, Q. and He, Y. – “Dynamic Distributed Genetics Algorithms”, Proceeding of Congress on Evolutionary Computation, pp 1132-1135, IEEE Service Centre, 2000.
- [14] Nanayakkara, T., et al. – “ Adaptive Optimization in a Class of Dynamic Environments using an Evolutionary Approach”, Journal of Evolutionary Computation, 7(1):45-68, 1999.
- [15] Morrison, R. W. – “ Designing Evolutionary Algorithms for Dynamic Environments”, PhD Thesis, George Mason University, USA, 2002
- [16] Morrison, R. W. and De Jong, K. – “A Test Problem Generator for Non-Stationary Environments”, Proceedings of Congress on Evolutionary Computation, VIII, pp 2047 – 2053, Washington DC, USA, IEEE Service Center, 1999.
- [17] Trojanowsky, K. – “Evolutionary Algorithms with Redundant Genetic Material for Non-Stationary Environments”, PhD. Thesis, Institute of Computer Science, Warsaw, University of Technology, Poland, 1994.